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A hypercube-based data structure for spatio-temporal exploration and analysis

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Abstract

This paper examines the opportunities existing within the multidimensional database (MDDB) approach to support spatio-temporal data exploration and analysis (*STEA*). MDDB-specific technology used for Data Warehousing and On-Line Analytical Processing (OLAP) is recognized for supporting easy query building, rapid query run times and simple interactive navigation within multiple displays, including maps. However, due to the lack of topological operators (spatial and temporal) in MDDBs limited *STEA* can be achieved. We thus propose a new data structure which implements a spatio-temporal dimension of topological operators in multidimensional databases. It involves a hierarchy of topological operators which uses the spatial and temporal relationships between instances of object classes. This hierarchy is structured according to granularity levels (4 in our example). Operators at the highest level carry general meanings while operators from the lowest levels have very specific meanings. This hierarchy also takes into consideration behavior models of spatial analysis in order to improve database usability. The topological relationships are described for each level of the hierarchy,

either through groups of operators or through the ISO/TC 211 9-intersection method (9IM) developed by [1] and the DE+9IM developed by [2]. These topological relationships are computed *a priori* and stored in the MDDB structure, often called hypercube, as 9 digit strings compatible with the ISO/TC211 topological operators [3]. This proposed solution represents a new approach for rapid and intuitive spatio-temporal analysis while being generic, software-independent and implementable in today's commercial MDDB engines.

1. Introduction

Spatio-temporal exploration and analysis are two complementary processes of Exploratory Data Analysis (EDA as defined by [4]). The *exploration process* aims at identifying hypotheses which are of interest to the user. These hypotheses consist of potential patterns, unusual occurrences or other types of relationships (*e.g.* causal, correlative, random, indirect). They help determine prediction rules and clusters that can be used to support decision-making. The *analysis process* determines the validity of hypotheses and in some cases proposes new ones. Such analysis determines the prevalence, the strength and the predictability of the hypotheses with the help of statistical analysis, data mining, visualization and report generators [5]. Exploration and analysis participate in knowledge discovery [6]. Knowledge Discovery in Databases (KDD) is a multi-step, interactive and often repetitive process to find new patterns in raw data. [7] defined KDD as the non-trivial process of identifying valid, novel, potentially useful, and ultimately understandable patterns in data. These concepts of data exploration, data analysis and knowledge discovery are central to decision-support systems. [8] described the heuristic process which results from the repetitive chain of exploration and analysis.

For the geographic context, this heuristic process has been studied by a number of researchers such as [9-13]. We define *STEA* as the collection of repetitive processes oriented either towards exploration or towards analysis which are involved in spatio-temporal knowledge discovery. Meteorologists make use of *STEA* to harness results from various climate models in order to synthesize computer model outputs and recognize patterns. For example, meteorologists examine the temperatures in the lower levels of the atmosphere, compare them across time to those at upper levels and hence determine the stability of air parcels. The analysis of weather trends result in an ensemble of predictions. These predictions help national weather services to produce detailed forecast reports and locate important weather events. These events include for example tornadoes, hurricanes, floods, flash floods, thunderstorms and winter storms. Detailed weather forecasts are relied on by a broad public, ranging from mariners, airlines, farmers, to the military and local meteorologists. Navigating back and forth from general views (coarse granularity) to very detailed views (fine granularity) according to varying themes, periods and regions implies repetitive large queries and fast answers. Important challenges arise with the complexity of running such queries against large volumes of data in reasonable time.

Building and running *STEA* queries for today's various flavors of databases and GIS configurations is a difficult task. In some cases, users lose their "train-of-thought" due to the complexity of the query language or the structure of the spatio-temporal database. Rather than focusing on the essence of their queries, they have to think on how to build them. In spite of better graphic user interfaces we believe this situation still prevails today. In other cases, users lose their "train-of-thought" due the long query run times. Such queries are both processor and storage intensive [14]. They cannot be planned in the context of *STEA* [6].

A closely related topic, Exploratory Visual Analysis (EVA) has emerged in the GIS community. EVA uses advanced visualization techniques to exploit a better understanding of the visual domain. However, EVA requires important training, expertise and data model awareness to adequately cope with computationally explosive solution spaces [15, 16]. In addition, not all users require advanced scientific visualization; most are satisfied with the capacity to navigate between multiple graphic displays which are one-, sometimes two-, but rarely three-dimensional [17]. Advances have permitted some progress in overcoming those barriers. For example, materialized views and sub-linear sorting [18] permit shorter query running times while real-time query follow-up [19] permits control over large queries. Natural structured languages [20], visual languages [21] and hybrid visual languages [22] help users to build their spatio-temporal queries.

Computer systems supporting *STEA* should permit the building of queries, either descriptive, graphic or cartographic without worrying about "how" this is achieved. Query results should be obtained in less than 10 seconds, which we believe is a reasonable time to maintain the users' "train-of-thought". In order to improve interactions with human users, they should also incorporate concepts or behavior models derived from the study of human spatial cognition [23]. [24] proposed eight rules for the design of multiple view systems. They promote the ability to use fast and easy to understand graphic displays (such as maps, tabular displays, pie charts, bar charts, line graphs, histograms, etc) within an environment which keeps the functionalities of conventional GISs.

In the actual context of growing availability of geospatial data and limited ability to explore and analyse this data we believe the MDDB approach is a promising avenue for *STEA*. It reduces the complexity associated with building and running queries and supports fast and easy to understand graphical displays. An example of the MDDB approach applied to *STEA* can be found in [25]. However, today's MDDBs implementations of spatio-temporal data lack explicit spatio-temporal operators and rely solely on the users' visual analysis to deduce spatio-temporal patterns.

2. The multidimensional database approach

The multidimensional database (MDDB) approach is a fundamental approach used for data warehouses, data marts, On-Line Analytical Processing (OLAP) and data mining. It supports the collection, storage, manipulation and reproduction of multidimensional data applications which are oriented towards supporting sophisticated analyses. It differs from the transaction-based approach typical of GIS in the sense that MDDBs are read-only subsets of data that have been imported from transactional databases, restructured and made available in a way that better supports decision-making. They are said to be analysis-oriented as opposed to transaction oriented. There exists several books and papers discussing the MDDB approach and an overview to spatial data can be found in [26].

Multidimensional databases (MDD) are made of hypercubes, dimensions, hierarchies and measures. A hypercube is a multidimensional array formed by the conjunction of several dimensions. The dimensions of a hypercube represent distinct categories of data used in analysis, *i.e.* themes supporting decision-making. For example, in a sales application, typical dimensions are products, time, locations and customers. Each dimension is organized into a conceptual hierarchy. Figure 1 presents an example of a dimension hierarchy for products of an industrial group. A dimension hierarchy is grouped into levels consisting of several dimension members. Each level in a dimension can be rolled together to form the values for the next highest level. For example, in the time dimension, days roll into months, and months roll into quarters. Measures are the quantitative values in the database that are analyzed. For example, for a sales application, typical measures are revenue, cost, units sold, discounts and returns. Figure 2 presents an example of a multi-dimensional query model featuring three dimensions ; product, time and location with their associated measures (*e.g.* revenue, cost, units sold, *etc*). Measures are analyzed against the dimension of a hypercube to produce multidimensional views of the hypercube. Usually, any multidimensional view (multidimensional query) can either be built or retrieved by an MDDB client in a few seconds and displayed through multiple types of graphical representations [17]. A typical multidimensional query for a sales application would be : how much revenue (the measure) did a company have last month (a member of the Time dimension) in Europe (a member from the Locations dimension) for all of its products (the top level of the Product dimension) and all of its customers (the top level of the Customer dimension). Figure 3 represents a multidimensional query applied to a hypercube. The potential of the MDDB approach has been largely recognized since a landmark paper by [27] in 1993 [17, 28, 29]. MDDBs have been implemented, used, validated and made profitable and the number of scientific papers dealing with MDDBs has grown significantly. MDDBs are recognized as an efficient solution for the exploration and analysis of data warehouses [30-32]. The MDDB approach has shown a good potential for efficient *STEA* [33, 34] and spatio-temporal knowledge discovery [26, 35, 36].

2.1 Cognitive compatibility of the multidimensional database approach

[23] proposed a virtual test bed which helps determine four broad classes of systems seeking to model some part of human spatial cognition. Based on characteristics of memory, processing, evolution, development, input and output his classification system shows how different types of computational system may be evaluated in their efforts to reproduce human cognitive behavior or to interface effectively with human cognitive systems.

Systems which do not attempt to reproduce human cognitive behavior through their internal processing structures directly but rather model such behavior in order to improve interactions with human users are said to be “cognitively compatible” [23]. In a sense, as the interface becomes “invisible”, users operate the system as a direct extension of their own reasoning capabilities [23]. This class of systems incorporates for example database designs anticipating human ways of understanding or viewing data and navigation systems providing instructions to help people navigate within new and unknown environments (*e.g.* speech-based car navigation systems) [23]. We believe the MDDB approach is a more compatible approach than other alternatives such as the ones defined by [23]. It supports most of the spatially related tasks required by the interpretation characteristics of cognition (although there are some other cognitive functions not supported). In this case, interpretation describes the ability to extract the concepts and references associated with an environment. The MDDB approach also supports several essential principles and processes of human cognition : categorization, hierarchies and information processing.

[37] and [38] have identified complex, multi-level relationships between grouped categories as an essential requirement for the exploration and synthesis of spatial knowledge.

Cognition relies on hierarchies for a broad variety of tasks, including learning [39], analysis [39], temporal ordering [40], temporal sequencing [41], language acquisition [40], organisation of information [42] and structuring of information [40]. There is evidence that spatial cognition relies on hierarchies to support representations of space [43-45]. [46, 47] have experimentally confirmed that cognitive geographic knowledge has a nested hierarchical structure.

The optimal time allowed for running a query has not yet been determined. [48] proposed the definition of a human temporal scale of action which is based on human cognitive modeling in computational layers or bands. He suggests that these computational bands emerge from the natural hierarchy of information processing as presented in Figure 4. This scale is divided into four bands according to physical and biological states : neural, cognitive, rational and social. [48] proposes that most human cognitive tasks occur between 10^{-1} and 10 seconds. This “cognitive band” can be divided on the basis of biological response times. Longer tasks belong either to the “rational band” (from 10^2 to 10^4

seconds) or the “social band” (from 10^5 to 10^7 seconds).

The short times required to either build or run queries through the MDDB approach are usually contained in Newell’s cognitive band. The MDDB approach therefore offers potential guarantees that its usability and performance do not interfere with the user’s on-going information processing during data exploration and analysis. [48]’s proposals are based on exhaustive experimentations (see for example [49]) and have received a wide acceptance in various fields. Confirmations of this 10-second limit can be found in results from studies which determine the ideal time to load an internet page [50-52], to request e-business services for mobile users [53] or to obtain successful responses from a server [51].

2.2 Going multidimensional

While GIS is possibly an important driver for the concept of a central data repository according to [54], it has not yet been adapted to the requirements which arise with a new era of information foraging. This era is characterized by the need to analyze trade-offs in the value of information gained against the costs of performing activity in human-computer interaction tasks [55]. These tasks cover activities such as assessing, seeking, handling and using information sources. Over the last two decades, spatio-temporal data has grown in volume and complexity at a rate driven by a thousand-fold increase in the ability to acquire data through sensors, experiments and computer simulation. Production costs are usually high. Satellites and supercomputer processing centers represent information sources that collectively cost billions. Unless effective ways to retrieve and manipulate spatio-temporal data are found, these expenditures will not yield the benefits to society that might be expected [56]. The ability to explore and analyse data is likely to become as important as computational modeling for solving problems.

The challenges arising in spatio-temporal exploration and analysis with conventional GISs cannot easily be overcome. Traditional GIS coupled to relational databases are capable of answering descriptive and spatial queries, often at high processing costs, through manipulations or languages ranging from simple to complex. Through our literature review, we found no exhaustive studies of the MDDB approach as a means to carry out *STEA*. [33] tested and confirmed its potential using a hypercube built with non-geometric spatial dimensions (without using map displays) [26]. Other researchers have used MDDBs for specialized and efficient indexing or processing, for example for spatial data mining [36, 57-59]. Examples can be found in diverse application domains: forestry [60, 61], agriculture [33], transport [34, 62], and health [63].

As the MDDB approach is neither genuinely built nor adequate for the storage and management of spatial data, it is not surprising that it has received little attention from the GIS community yet. Meanwhile, as time and space are intrinsic dimensions for business problems, the MDDB approach has been gaining attention from the business world [64-66]. Indeed time and space often lie at the core of business intelligence aiming at defining accurate demographic profiles of customers or clients. Such profiles help for example to determine lifestyle segments which are an essential tool to understand if a company is in danger of losing customers to the competition or for saving on postage and mailing costs. Customer satisfaction surveys can also be linked to lifestyle segments to identify areas where the company is not adequately servicing its customer base.

Traditionally, multidimensional database clients-server solutions could not permit the usage of maps to graphically display data. New clients like Pro-Clarity (www.proclarity.com) and Cognos Visualizer (www.cognos.com) demonstrate that a mapping display is gaining support in the industry. Whereas it was already possible to carry out *STEA* with conventional clients without mapping displays [33] these mapping capabilities now amplify existing opportunities. In spite of an interesting potential for spatio-temporal exploration and analysis today's solutions cannot permit adequate spatio-temporal reasoning because they lacks metric, directional and topological operators (either spatial or temporal). Without topological operators, the exploration and analysis of spatio-temporal data is restricted to dimension and measure browsing. End users have to navigate around dimensions hierarchies visually seeking spatial relationships between members, levels and dimensions. Hence only limited *STEA* can be undertaken. Topological operators can only be found in traditional GIS software and in some spatially enhanced object-relational databases. Stored as algorithms or in enriched data structures, they are either processor or storage intensive. Temporal topological operators are not available in conventional GIS software. For relational databases, they can be implemented through query languages but suffer from excessive complexity. Spatio-temporal operators simply do not exist in conventional GIS products, they can only be obtained through the combination of results from separate spatial and temporal topological queries.

3. Topological operators

3.1 Importance of topological operators for STEA

While geographers have always intuitively understood the "general theory of spatial relationships" [67] it is only recently that researchers have gained a better understanding. This progress has been made possible by advances in three domains : geometric relationships among spatial objects, the theory of spatial representation (see [68]) and spatial cognition (see [69]). Explicit representations of geometric relationships among arbitrary spatial objects have been exhaustively studied (see for example [1, 2, 70]). These relationships require cognitive structuring in order to be adequately manipulated by human subjects [71].

In geographic space, topology is considered to be first-class information, whereas direction and metric properties are used as refinements that are frequently less exactly captured [72, 73]. [74], based on the experiments of [75] and [76], proposed topology as the essential foundation for spatial knowledge acquisition for map and navigation learners. In a regular environment, a map-learner acquires topologic and metrical configurational knowledge at the outset, whereas a navigation learner initially acquires route knowledge, and with increased experience in this environment, acquires metrical configuration knowledge [74]. In an irregular environment, a map-learner acquires topologic and metrical configurational knowledge at the outset, whereas a navigation learner initially acquires route knowledge of this environment, but does not acquire topological and metrical configuration knowledge [74]. While the test conditions and aims of [75] and [76] experiments were different, these assertions are supported by other studies which report similar results [77-82]. While geographic space and time are tightly coupled [72, 83] no conventional GIS has been known to offer either temporal [84-86] or spatio-temporal topological operators (when they do, they are limited to database time and offer only limited functionality).

Topological relations appear to be essential for human information processing. We believe it is also the case during *STEA*. Spatial and temporal topological operators should therefore be available to *STEA* users and their operation should not interfere with their thinking process. Futhermore, while there exists a finite number of topological operators, there is little chance that a typical user understands, needs and makes use of their subtleties. Consequently, topological operators must exist with general definitions as well as with detailed definitions in order to satisfy all levels of STEA, ranging from coarse to detailed analysis.

3.2 A hierarchy of spatio-temporal topological operators

Even if a complete set of operators could be implemented, there is a need to simplify their inherent complexity. [71] tested human subjects in order to determine models of spatial relations between lines and regions. Their results suggest that many spatial relations can be represented by particular subsets of the primitives differentiated by the representation of geometric relationships. This trend has also been confirmed by [73]. Such experiments have not been carried out for other spatial relations yet (lines & lines, regions & regions, points & points, points & lines and points & regions) but we can expect similar results as earlier studies seem to indicate [47, 87-92]. In a research project involving users at Natural Resources Québec, [93] have illustrated the need for grouping relations in order to facilitate their understanding by users. They built generalizations on top of the 9IM proposed by [1].

The agreement diagram reached by over half of the subjects of the two experiments discussed by [71] could help determine probable groupings of topological operators between linear and region features. A hierarchy with different levels of granularity can be organized to permit grouping of topological operators into meaningful categories. Such categories would permit users to manipulate multiple levels of refinement of topological operators. For example, in order to get a broad picture of the number of states that intersect a national highway, a user could start by selecting the operator “intersect” to produce the first cross-dimensional view. As finer detail is required, the user can drill down in the topological operator dimension by selecting the macro-level operator “interior intersection” in order to produce a second cross-dimensional view. If even greater detail is required, the user can drill down further to the meso-level to select one of six possibilities of interior intersection. Finally, if the user is interested in the dimensionality of the intersection, he can drill down to the micro-level in order to choose from the possibilities which are relevant to his previous selection.

Spatio-temporal queries addressed to a system usually combine topological operators in both the spatial and temporal dimensions. For example, a query requesting the list of people that have been at the same place at the same time could be expressed by an 'equal to or intersect' operator for space and by a 'equal to or intersect' for time. They could well be conceptually combined in a unique 'same time, same place' operator. Such an operator carries a precise spatio-temporal meaning [72, 83]. [94] have proposed a data model and query language capable of handling time-dependent geometries including those associated with moving objects. Their conceptually clean foundation offers precise guidelines for implementing a spatio-temporal database DBMS extension.

It would be possible to integrate a hierarchy of spatio-temporal topological operators within a user interface. We propose to integrate it within a multidimensional database as a specific dimension. Through its intrinsic support for hierarchies, exploration and analysis (see section 4 & 6) the MDDB approach offers a unique opportunity to integrate and efficiently exploit a hierarchy of spatio-temporal topological operators. MDDBs could therefore play an essential role in *STEA*. Used as filters, members of the hierarchy of spatio-temporal topological operators permit users to isolate portions of the multidimensional database which satisfy topological constraints instead of navigating across space and time dimensions for hints about spatio-temporal queries.

Quite a few MDDBs already deal with dimensions expressing forms of spatial and temporal dimensionalities [64, 65]. Through taxonomical arrangement in nested hierarchies [38], some forms of topological relationships are expressed between the members. For example, the “entirely contains” operator could be inferred from the relationship between “country” members and “province” members which is typical of geographical dimensionalities.

3.3 Structure of the proposed spatio-temporal topological dimension

We propose to incorporate two approaches to the description of topological relationships in a hierarchy, namely that developed by [1], the 9-intersection model (9IM), and that developed by [2], the DE+9IM. Present at different levels of granularity in our hierarchy, each approach is included in the ISO/TC211 standards [3]. They both express the relationship between two elements through a 3x3 matrix. The elements to be analysed by these two methods can be a point, a line or a polygon. [1] describe the relationship through existing (1) and non-existing (0) relations between sectors of the element (inside, border and outside). [2] follow a similar approach but precise the dimensionality (either 0d, 1-d or 2-d) of sectors intersections in order to describe each relationship in the DE+9IM. Figure 5 presents the generic form of these two models. Figures 6 to 8 illustrate how they differ from each other in two specific examples. The symbol "°" designates the interior portion of the element, "d" designates the boundary of the element and "-" designates the space outside the element.

In order to permit multiple levels of refinement for topological queries, we propose a hierarchal architecture structured on different granularity levels of topological operators. We call it the spatio-temporal Topological Operator Dimension (TOD) architecture. Figure 9 presents the spatial, temporal and spatio-temporal hierarchy subsets of this dimension. We propose to combine spatial and temporal topological operators in order to meet specific spatio-temporal conditions. For example, at the first level, the spatial and temporal "intersect" topological operators can be combined into a "same time, same place" spatio-temporal topological operator. Similarly, the combination of the spatial operator "intersect" with the temporal operator "disjoint" can result into a "different time, same place" spatio-temporal topological operator. Our goal in this paper is not to define an exhaustive list of spatio-temporal topological operators but rather to illustrate how they can be structured and used. Such operators and their underlying data models have been discussed by [95] and by [94] in the context of moving objects. Figure 10 presents a business query example implying the Topological Operator Dimension to isolate the stores locations (polygons) adjacent in space and over-lapped-by in time to trucking delivery routes (lines) in Belgium for milk products during the third quarter of 2001.

The first level of the TOD corresponds to the highest level of aggregation of topological operators. This level has three members : “any time, some place”, “some time, any place” and “some time, some place”. The “any time, some place” operator corresponds to the highest level of aggregation of spatial topological operators. The “some time, any place” operator corresponds to the highest level of aggregation of temporal topological operators. The “some time, some place” operator corresponds to the highest level of aggregation of combinations of spatial and temporal topological operators.

The second level of the TOD corresponds to a simple distinction between “same” and “different” for each member of the first level. While “same” can have multiple meanings (*e.g.* “intersect” or “equal” or “adjacent”), “different” always carries the meaning “disjoint”. In the context of *STEA*, we feel that this distinction should be made as soon as possible.

The third level of the TOD corresponds to a refinement of each of the members of the second level. The dependent members of “any time, same place” and “any time, different place” correspond to the highest aggregations of spatial topological operators according to [96]. The dependent members of “same time, any place” and “different time, any place” correspond to temporal operators proposed by [97]. The dependent members of “same time, same place”, “same time, different place”, “different time, same place” and “different time, different place” correspond to the multiple combinations of the third level operators from [96] and [97].

The next levels correspond to multiple refinements of either the spatial or temporal topological operators. These refinements are first made according to the topological operator groups determined by [96], then to the 9IM and finally to the DE+9IM. Only a relevant subset of TOD is made available at once to users depending on the geometry of their selection of spatio-temporal entities. A spatio-temporal entity is formed by a spatial entity and an associated temporal primitive. For example, if a user is interested in spatio-temporal topological relationships between a river (1-d) and a county (2-d) only the TOD operators related to lines and polygons will be offered.

For the descendants of the first level "some time, some place" operator, the hierarchy is structured in such a way that it is always possible to maintain a separate level of granularity between the spatial and temporal constrains. [98] have demonstrated that a temporal order of presentation influenced estimates for bearings, distances and positions. [99] demonstrated that temporal distance influenced distance estimation only under the spatially close condition. Both these results contribute to the temporal priming effect in spatial memory and show how subjective representations of spatial relations may be modified according to temporal conditions of acquisition. The TOD is presented here in a generic form which includes all possible combinations of granularities of spatio-temporal topological operators. Due to the limitations of the actual state of research in the field of natural-language spatial relations, the TOD subset offering cognitive aggregations of topological relationships is limited to lines and regions. The only study in this field was carried out by [71]. It focuses on cognitive aggregations of topological relationships for lines and regions.

Our approach is generic and can use any hierarchy of operators. Thus, we invite readers to build their own aggregations and to implement them through our approach. At the highest level of the TOD hierarchy, the separation of the three possible domains of topological relationships (spatial, temporal and spatio-temporal) permits immediate isolation of the adequate topological constraints to be applied to the multidimensional query. These operators capture in a simple and meaningful expression a coupling of geographic space and time which [72, 83] have highlighted as an essential component of human spatio-temporal reasoning. A good example of such couplings can be found in graphic timetables for railroad lines in [100]. It can also be found in the definition of the agrarian unit "arpent". Defined as the surface which twelve men can work in a single day, this unit combines both spatial and temporal meanings. Figure 11 presents the TOD members and hierarchical relationships for the spatial, temporal and cognitive subsets.

In order to permit adequate temporal and spatio-temporal reasoning, it is essential to provide users with temporal models which support both representation and reasoning. Such models usually resolve problems which are either symbolic or mixed (numeric and symbolic) within the framework, for example, of [97]'s interval calculus. There exists other approaches for building such temporal models *i.e.* constraint propagation and logic and neuronal networks. For the TOD, we use [97]'s interval calculus approach. The same approach which was presented for the spatial axis is also to be applied to the temporal axis. After plotting the events associated to the spatio-temporal entities on a time axis, either as points or lines, topological and ordering queries can be executed in order to fully describe the topological relationships between all the events associated with spatial entities. [101] used this technique and demonstrated that it is well suited for spatio-temporal analysis. While this method permits proper temporal analysis of static phenomena, it lacks the flexibility required for dynamic phenomena. A workaround could be found by superimposing the timelines along the path taken by the dynamic phenomena to be studied. This approach of spatio-temporal paths is inspired by the methods proposed by [102] and [103].

4. Implementation in a MDDB hypercube

As the preparation of data for its import to a MDDB varies among software packages, there is no straightforward way of integrating the TOD in a MDDB. What this paper proposes is to build two tables that all MDDBs can import either as flat text files or as linked tables through a conventional ODBC connection. The goal of this paper is not to define an ideal preparation of the import table but rather to illustrate how it can be done and the advantages such a formulation carries. The first table, the TOD table, contains the TOD members and their hierarchical relationships. The second table describes topological relationships between spatial entities and between temporal primitives. These two tables must be integrated with other descriptive tables to form a hypercube or can become a stand-alone hypercube which is shared between standard hypercubes in the MDDB. We propose to use the DE+9IM to specify the 9IM and the other aggregations presented in the previous section. From the 3x3 matrix of these two models, the nine values are extracted in a row-major form to compose strings of "MatrixValues" :

{v11, v12, v13, v21, v22, v23, v31, v32, v33}

This method is based on the ISO TC-211 standards [3]. They introduce two topological operators, eRelate and cRelate which are respectively based on the 9IM and the DE+9IM. These operators are used to test topological relationships between objects. They are used as follows :

eRelate(ObjectA,ObjectB,MatrixValues) & cRelate(ObjectA,ObjectB,MatrixValues)

They return either a 1 or 0 to validate or invalidate the topological relationship between A and B expressed by "MatrixValues". Neither of these operators is implemented in contemporary GIS or object-relational databases with spatial extensions. In some cases, subsets of the relationships expressed by eRelate and cRelate are available through customized topological operators (e.*g.* in Intergraph

Dynamo and MGE) or fixed names (*e.g.* in Oracle Spatial). Strings of “MatrixValues” from the 9IM and the DE+9IM act as unique identifiers for the topological relationships which they specify. These strings are the respective members of the last levels of granularity of the TOD. Figure 11 presents the spatial, temporal and cognitive subsets which are used in conjunction to form the levels and members of the spatio-temporal subset TOD. Figures 12 and 13 show the resulting topological strings for both intersection models applied to the previous two examples. The second table describes topological relationships between spatial entities and between temporal primitives. Figure 14 illustrates how the relationships can be encoded in a table which stores the topological strings at the finest granularity. The size of this table is likely to grow in size with the number of spatial entities and their associated temporal primitives. We propose not to store spatial disjoint relationships to reduce the table size. If no relationship exists between two spatial entities in the second table it is assumed they are disjoint.

As there is no straightforward to compute all topological relationships between spatial entities and between temporal primitives according to either the 9IM or the DE+9IM we propose a two-phase query model. In the first phase, a spatial index is built by a spatial database such as PostGIS for PostgreSQL, Oracle Spatial, ESRI SDE or spatial datablade for Informix. This spatial index insures that entities which are near in the coordinate space are near in the ordering. Minimum bounding rectangles (MBRs) are tested for intersection using the spatial index to filter out the obvious cases. MBRs are coarse approximations and do not guarantee intersection of actual spatial objects. Testing their intersections permits to highly reduce the amount of exact computations to be carried out for most common spatial datasets where most features do not overlap. In the second phase, exact computation of topological relationships is carried out between the entities which are likely to interact by a software such as the Java Topology Suite (JTS). The Java Topology Suite is a free Java API (www.vividsolutions.com) of spatial predicates and functions based on the ISO/TC211 standards and simple features specification for SQL from the Open GIS Consortium. The topological operators used in the second phase should always be of the finest granularity possible with respect to the TOD in order to facilitate later aggregations.

This processor and disk intensive operation is the only disadvantage of the integration of the TOD within a MDDB. Nevertheless, it is also typical of the data feeding process of multilevel data warehouses and poses no specific issue not addressed by data warehousing. There are other approaches to avoid the N^2 space requirement associated with the computation of topological relationships between spatial entities and temporal primitives. One would be to use computational geometry algorithms such as the *plane sweep* algorithm [104] to determine intersections in the first phase of the query model. Other can be found, *i.e.* by exploiting the explicit topological data structures of some GISs or taking advantage of topologically enriched data structures such as the Voronoi diagram [105]. Techniques similar to efficient polygon amalgamation methods [106] could certainly improve the running time of this type of operation. It is beyond the scope of this article to determine the adequacy of different approaches for datasets varying in size and complexity.

In our experimentation, we used a dataset made of 29.475 linear objects corresponding to the linear interpolation between successive positions of radio amateur mobile stations broadcasted through the Automatic Position Reporting System (ARPS). On the spatial side, this dataset covers the cities of New York and Washington. On the temporal side, it spans from August,13 2001 to November 12, 2001. Oracle Spatial was used in the first phase of our query model and Java Topology Suite was used in the second phase. In our experimentation, we used a 1.7 GHz Pentium 4 computer with 768 Mb RAM. It took 6 seconds to compute a spatial index (R-tree) for the 29.475 spatial entities. We then used the SDO_FILTER command to identify the pairs of spatial entities which were likely to interact. This filtering took 11 minutes. 14.834.713 pairs of entities were identified as likely to interact. The complexity of SDO_FILTER is believed to be around log n. Using the Java Topology Suite, we computed all the relevant topological relationships between the identified spatial pairs in 10 hours. It took 1 hour to compute all temporal topological relationships between the temporal primitives associated with the spatial entities which were likely to interact. We carried out a similar test with a larger dataset (853.764 spatial entities) using the same approach. It took 11 minutes to build a spatial index with Oracle Spatial and 51 hours to identify the 1.422.288.743 pairs of spatial entities which were likely to interact through the SDO_FILTER command. A homemade software carried out the same calculation in 38 minutes on the same computer. This software used a fixed tile index and determinant calculation.

The table used to store the explicit topological relationships between pairs of entities (spatial or temporal) can become fairly large. However, like all facts inserted in a MDDB, it will be compressed by a factor usually varying from 20-to-1 to 10-to-1 in a hypercube. This compression feature is built-in into most commercial MDDB servers and its structure is proprietary. [107] have tested the compression of a six-dimensional banking hypercube based on a 13 million-row fact table using Microsoft Analysis Services 2000. The source relational fact table size was 5188 Mb with indexes but no aggregates. The resulting hypercube size was 336 Mb even when including a significant number of aggregates.

Once these two tables have been prepared, relations can be created either *a priori* or *a posteriori* to the hypercube import process depending on the MDDB import mode. The field describing the topological relationship in the second table is joined through a many-to-many relation to the appropriate level from the first table. The fields describing the objects implied in the topological relationship in the second table should be joined through a many-to-one relation with their counterparts in descriptive tables if applicable. Optimisation techniques are beyond the scope of this paper but have been addressed by the primary author in his PhD research. Once the optimization complete, we expect to be able to use the

TOD in conjunction with other dimensions to explore and analyse spatio-temporal data with spatio-temporal topological constrains (either cognitively compatible or based on the ISO/TC211 standards).

5. Concluding remarks

This paper introduced the multidimensional database (MDDB) approach as an innovative way of supporting *STEA*. It identified, highlighted and extended this interesting solution to the *STEA* problem and suggested a result better geared for decision making than applications built with the transactional approach. While KDD research is rapidly advancing, we are still far from fully automated systems. In order to make profitable the important investments made in spatio-temporal data collection, processing and storage, we believe that, at this stage, it is essential to take advantage of human expertise. Systems designed to support *STEA* must therefore conform with at least some essential human cognitive capabilities to permit their efficient usage. In the field of data exploration and analysis, the required time and skills to build and run a query are critical. In this paper, we suggested that a 10 second period appears to be a reasonable time to maintain a train-of-thought. We also highlighted the potential of the MDDB approach to support such interaction and performance without any query language requirements.

The key contributions of this research were to propose a hierarchical MDDB structure of topological operators based on multiple levels of granularities which incorporate both spatial and temporal topological relations. This research also introduced an implementation solution for these concepts. The result is a simple, innovative and promising way of supporting *STEA* for KDD. Considerations such as the combination of topology with direction [108], topological equivalence and similarity in multiple-representation geographic databases [109], approximate topological relations [110] and spatio-temporal patterns [111] are not discussed here despite their relevance in spatio-temporal reasoning and obvious potential to be supported by our approach.

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